On the Theory of Experiments to detect Aberrations of the Second Degree

LXI. *On the Theory of Experiments to detect Aberrations of the Second Degree. By Edward W. Morley, Ph.D., LL.D., Professor in Western Reserve University, and Dayton C. Miller, Ph.D., Professor in Case School of Applied Science, Cleveland, Ohio*[1].

[Plate IX.]

IN 1887 Michelson and one of the present writers made an experiment "On the Relative Motion of the Earth and the Luminiferous Aether".[2] We found that, if there were any effect, it was not sensibly larger than one-fortieth of the amount expected. To explain this result, FitzGerald and Lorentz suggested that the motion of translation of a solid through the aether produces a contraction in the direction of the drift, with extension transversely, the amount of which is proportional to the square of the ratio of velocities of translation and of light.

Such a contraction can be imagined in two ways. It may be thought to be independent of the physical properties of the solid and governed only by geometric conditions; so that sandstone and pine, if of the same form, should be affected in the same ratio. On the other hand, the contraction may depend upon the physical properties of the solid; so that pine-timber would doubtless suffer a greater compression than sandstone. If the compression annul the expected effect in one apparatus, it may in another apparatus give place to an effect other than zero, perhaps with the contrary sign.

We have now completed an experiment in which two different pine-structures have been used, and in which the optical parts have been so enlarged as to produce an effect 2.3 times as great as the apparatus of 1887. The object was to determine whether there is any difference between the behaviour of sandstone and of pine.

When Michelson and Morley got a null result in 1887, it was thought sufficient to give the theory for merely the maximum and the minimum expected in the four principal azimuths, without mention of the phenomena at intermediate azimuths. The theory also neglected powers higher than the second of the ratio of the velocities. Recently, Dr. Hicks[3] has published a profound and elaborate discussion of the theory, obtained by methods which are not approximate.

He develops expressions for angles of reflexion, for wavelength after reflexion, and for the conditions which determine the network of parallelograms formed by the two systems of wave-fronts. The diagonals of these parallelograms are alternately lines of maximum and minimum disturbance in the aether, so that they define the interference phenomena. These expressions are not only rigorous, but also general, applying to any adjustment whatever of the optical parts of the apparatus, and form a welcome contribution to the thorough understanding of the theory of the Michelson and Morley experiment.

In one passage he says that a term added by him "may entirely modify the nature of the changes produced as the direction of the drift alters" ; and some appear to think that the inference from the earlier experiment is involved in doubt by this discussion. It is therefore well to examine again the theory.

Let D, Pl. IX. fig. 1, be a plane-parallel glass plate, silvered so thinly that equal quantities of light are transmitted and reflected. S being a source of light, part of the light passes through D, moves on to the

plane mirror II, where it is reflected back to D; here, part is transmitted and lost, part is reflected to the observer at T. The other part of the entering light is reflected at the first surface of D, reflected again by the mirror I, is in part reflected by D, and lost, in part transmitted through D, and proceeds towards T. If distances and angles are suitable, the reunited rays between D and T will produce interference phenomena. If distances are equal, we may obtain interference phenomena in white light. In one of the usual adjustments of distances and angles, parallel fringes are seen when the eye or the telescope is made to give distinct vision of one of the mirrors I or II. The fringes apparently coincide with these surfaces. A central fringe is black; on either side are coloured fringes, less and less distinct till they fade away into uniform illumination. If the path of either ray is shortened, the fringes move rapidly to one side. If we engrave a scale on I or II, we can, after any alteration of one of the paths, restore with great accuracy and ease the former relations by bringing the central dark fringe to its original place on this scale. If the motion of the earth through the aether were the cause of this change of path, we could measure the amount of change by measuring the displacement. Suppose, fig. 2, that the apparatus moves in the direction of the arrow through the stationary aether. While the ray of light moves from D to I and returns to D, the mirror D moves to the new position D_2. The angle of reflexion from D is no longer equal to the angle of incidence. The ray moving from D towards II finds the latter in the position II_2, returns to D_2, and is reflected from D_2 nearly in the same direction with the ray from I. In four azimuths of the apparatus the coincidence is exact; for all others, the ray I and the ray II are inclined at a small angle which, at its maximum, is numerically equal to v^2/V^2, v and V being the velocities of the apparatus and of light. Since the angle δ, the total aberration, cannot be observed, being annulled by the motion of the observing telescope at T, we can hope to detect merely this aberration of the second degree, namely, the small angle between the emergent rays I and II.

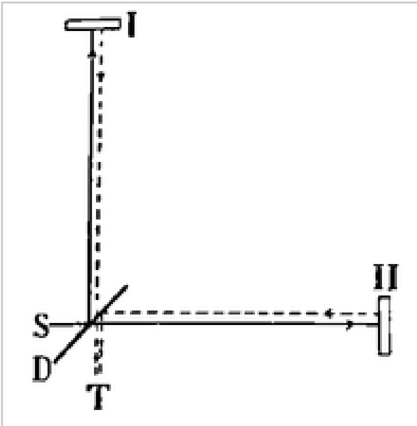

Fig. 1

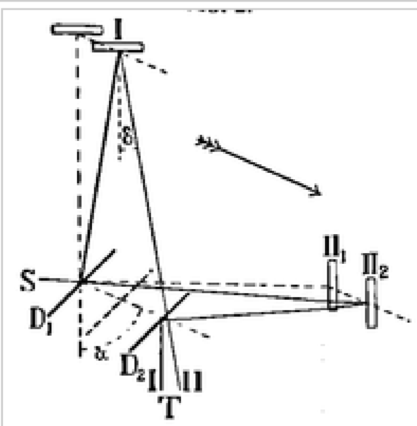

Fig. 2

With the adjustments just supposed, there are four methods of measuring interference phenomena which in turn measure the angle sought. We may use a micrometer in the telescope, or a scale engraved on I or on II; we may use mechanical compensation to return a displaced fringe to its marked position, or we may use optical compensation.

In another adjustment the fringes are made infinitely broad. We are then limited to the last pair of methods. This pair, especially the last method, is capable of very great precision. When Michelson and Morley set up the first apparatus in which they utilized this method, the mean error of a setting, in which the observer did not himself see the reading, was less than the two-hundredth part of a wavelength. Since the theory of the apparatus in this special case is simpler, the discussion will assume this adjustment.

Accordingly, let the angles I B D, II B D, fig. 3 (PI. IX.), be equal to each other and to 45°. Let the three planes intersect in a common point B. For brevity, imagine that the mirrors themselves are produced so as

to intersect in this point. Assume that the system is moving through the aether in a direction making an angle of 67½° with the direction of the light entering the telescope, as indicated in fig. 2. The velocities o£ the apparatus and of light being denoted by v, V, assume that v/V=1/5.

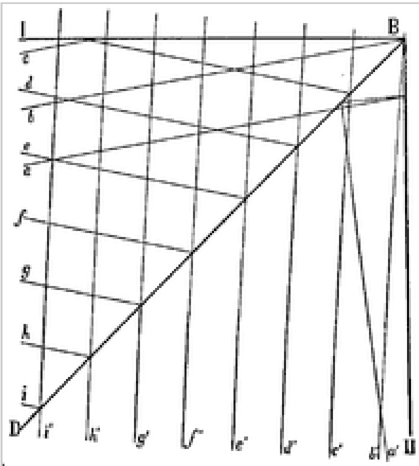

Fig. 3

A certain wave-front enters the apparatus, making with II an angle which is to be specified. If some given ray enters the apparatus so as to pass axially through the telescope, rays making an angle of 5 minutes on either side of it will pass through our actual apparatus. Almost any ray, wisely selected, may be used to determine what we desire to know about the whole pencil. For instance, we might select the ray which, after reflexion from II, shall return to the point in the mirror D at which it first passed through this semi transparent mirror; or the ray which, after reflexion from I, returns to D with the same exactness. The simplest treatment is possible when we select that system o£ wave-fronts which make with the mirror II an angle $\sin^{-1}\frac{v}{V}\cos\alpha$, where α is the azimuth of the apparatus measured from the position in which its motion through the aether is parallel to the axis of the observing telescope. The azimuth of the motion assumed in figs. 2 and 3 is 67° 30'.

We will now examine the condition of the wave-fronts in the apparatus, fig. 3, at two specified instants, using two diagrams to avoid confusing the numerous lines. In fig. 3 are shown nine wave-fronts making the specified angle with II. The wave-fronts of the transmitted fraction are denoted by accented letters. Seven have not yet reached the mirror II; *b′* intersects II in the common point B; *a′* has been reflected from II, and its upper part begins to he reflected from D. At the same instant are shown the wave-fronts of the other system by unaccented letters. All have been in part reflected from D; *c* begins to be reflected from I; *b* intersects I in the common point; *a* is quite cleared from the mirror I. In fig. 4 we follow the same nine wave-fronts. Of the transmitted waves, *a′* has quite cleared the system of mirrors; *b′* is just clearing D; five are but partly reflected from D; *h′* passes through the common point B, and is beginning to be reflected by D; *i′* has not yet reached II. Of the other system of waves, *a* and *b* have cleared the system of mirrors; five are passing through the semitransparent mirror D; *h* passes through the common point B, having just finished its reflexion from I; *i* is just beginning this reflexion. For all azimuths except four the general conditions are those of the diagram, but the amount and direction of the various inclinations alter with the azimuth.

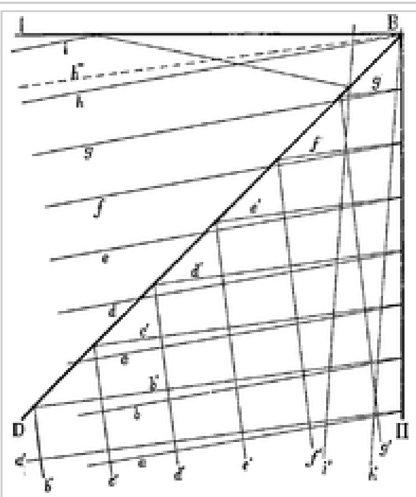

Fig. 4

The wave-front *h* is established in its whole length when it passes through the common point. The wave-front corresponding to it in the other system is, this instant, infinitely short, and *a′* is the first to be established in its entirety. But the position of a fictitious wave of this system, *h″*, is determined by two conditions — first, that it be parallel to *a′,* and, secondly, that it coincide with the infinitely short wave-front *h′* at the common point B. Except at four certain azimuths, these two wave-fronts, in the same phase. and intersecting in a common point B, will be inclined to each other at a small angle. To measure or to detect this inclination is to measure or to detect the secondary aberration which interests us. If we could measure the perpendicular distance between these wave-fronts at a sufficient distance from B, we should know the angle between them. But *h″* is only a fictitious line. What we cannot measure between *h* and *h″*

we can measure between a and a', provided we can determine the point of intersection between a and a', and provided this be found in a convenient position. We have therefore to determine the point of intersection of a and a', knowing that of h and h''.

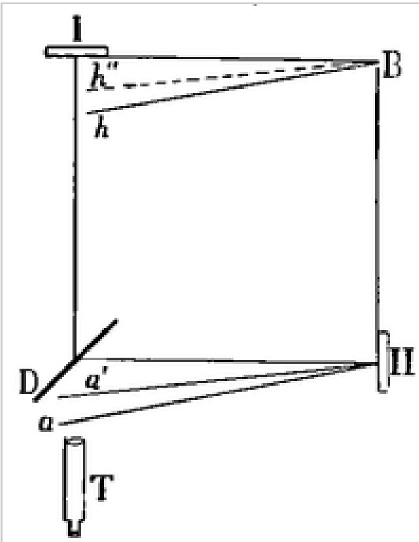

Fig. 5

The observing telescope is shown at T, fig. 5. Its axis is parallel to B II. We will show that the phase-difference of a and a' is constant at all points on any line parallel to the line B II, or to the axis of the telescope.

If we write λ, λ', *not for wave-lengths*, but for the perpendicular distance between consecutive wave-fronts of the same phase, and δ, δ' for the total aberration of the wave-fronts of the two systems, we have to show that $\frac{\lambda'}{\cos\delta'} - \frac{\lambda}{\cos\delta}$ is identically zero for eight specified equidistant azimuths, and is not greater than O.3 $\frac{v^4}{V^4}$ for other azimuths. Each of these quantities is determined by a complicated expression; and the equality specified can be most readily determined by trigonometric computation.

To prove the proposition, therefore, we will take that azimuth where, according to Dr. Hicks, the shifting of the intersection is a maximum, and we will assume the extreme case where the velocity of the apparatus is half that of light.

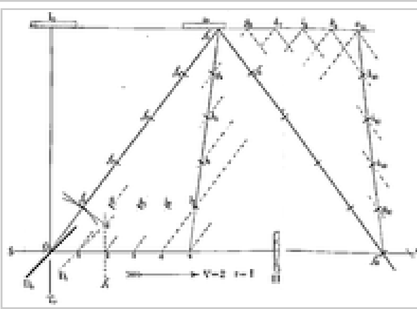
Fig. 6

In fig. 6, the mirrors D, I, and II are accordingly supposed to move in the direction of the arrow. Let π be the period of the waves of light incident on D; according to the previous specification, the angle between these wave-fronts and the plane of II is $\sin^{-1}\frac{v}{V}\cos\alpha$; that is, they are parallel to II. Lay off on c d, the line of motion of a certain point of the mirror D, the positions of this point at the times 0, τ, 2τ, &c. Positions of D and of I and II at certain times are also noted in the same way : all numerical subscripts denote times. The source moves with the apparatus, and therefore, with the assumed ratio of velocities, the apparent wavelength of the light incident at D is half the wave-length in the case of rest, and is half the distance described by a wave-front in the unit of time. Let the initial position be one in which a wave-front passes through the given point in the mirror and through the point 0 in the line of motion. At the time t=τ the mirror is at 1, and the wave-front in question cuts the line of motion in 2, and intersects the mirroring. The wave-front reflected from D at t=0 will have reached the point f_1, and the tangent ef_1 establishes the reflected wave-front. At the times 2τ, 3τ, &c., this particular disturbance will be found at f_2, f_3, f_4, &c.

When D is at a position 2, a new disturbance will have been established at g, which, at the time $t=5\tau$, will be found at g_5. In the same way, h_5, i_5, k_5 will have arranged themselves in the line f_5k_5. At the time $t=10\tau$ the six wave-fronts will have been reflected from I, and will be placed along the line $o_{10}f_{10}$. The angle f_{10} O I o is equal to the aberration of the wave-fronts after reflexion from D. As, at this azimuth, the angles of incidence and of reflexion at I are equal, this angle is also the aberration of the emergent rays.

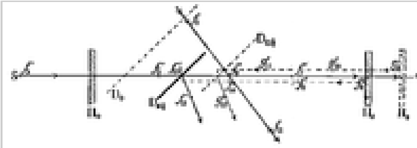

Fig. 7

Part of the wave-front f, indicated by f', will be transmitted through the mirror D. It will overtake the mirror II at the time $t=8\tau$, when II will have position marked II_8, fig. 7. Returning after reflexion, it will take the position noted for the times $t=9\tau$, $t = 10\tau$; and meeting the mirror D at $t=10\frac{2}{3}\tau$, it will be reflected as shown f'_{11}, at an angle whose tangent is given by the formula below. The following wave-front g' will be reflected one period later, at II_9, and it is shown in several positions. The wave-front f, belonging to the other system, having passed through the mirror, and having reached the line, Sd, at $t=10\tau$, is shown at f_{11}. In fig. 8 is shown the position of the wave-fronts below the mirror D for the time $t=15\tau$. f_{15}, and f'_1 have moved along the paths indicated, while the other wave-fronts have moved in a corresponding manner, their position at the time $t=15\tau$ being as shown in the figure. The wave-fronts of the unaccented system are placed on the line op; the aberration of the system is equal to the angle δ. The wave-fronts of the accented system are placed on the line qr; the corresponding aberration is the angle δ'; the line $\mathrm{T}_{15}q$ being the position of the axis of the observing telescope at the time $t=15\tau$. Produce the planes of the wave-fronts, draw line l, l', parallel to $q\mathrm{T}_{15}$, the axis of the telescope, each terminated by the planes of consecutive wave-fronts. Their lengths are $l = \dfrac{\lambda}{\cos\delta}$, $l' = \dfrac{\lambda'}{\cos\delta'}$. It is to be proved that $l=l'$.

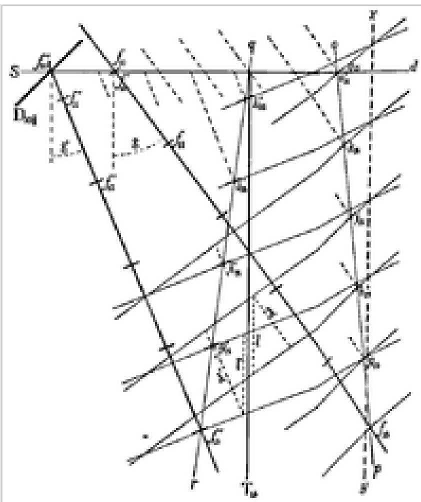

Fig. 8

Putting φ for the angle of incidence and φ' for the angle of reflexion, we have for both reflexions at D, $\varphi=45°$. For this azimuth there is no change of angles at I and II. The angles of reflexion are given by the equation

$$\tan\frac{\phi'}{2} = \tan\frac{\phi}{2}\cdot\frac{\mathrm{V}-u}{\mathrm{V}+u}$$

where u is the velocity of the surface perpendicular to its own plane; negative, if moving away from the approaching light. *With the assumed adjustments*, we therefore have for the reflexion of the unaccented system at D:

$$\frac{\sin\ 45^\circ+\sin\delta}{\cos\ 45^\circ+\cos\delta} = \frac{\sin\ 45^\circ+\sin 0^\circ}{\cos\ 45^\circ+\cos 0^\circ}\cdot\frac{2+\sqrt{0.5}}{2-\sqrt{0.5}}$$
$$= \frac{\sqrt{0.5}+0.6}{\sqrt{0.5}+0.8};\ \therefore \cos\delta = 0.8.$$

For the reflexion of the accented system we have:

$$\frac{\sin\ 45^\circ-\sin\delta'}{\cos\ 45^\circ+\cos\delta'} = \frac{\sin\ 45^\circ-\sin 0^\circ}{\cos\ 45^\circ+\cos 0^\circ}\cdot\frac{2-\sqrt{0.5}}{2+\sqrt{0.5}}$$

$$= \frac{\sqrt{0.5} - \frac{5}{13}}{\sqrt{0.5} + \frac{12}{13}}; \therefore \cos\delta = \frac{12}{13}.$$

By equation (3) of Dr. Hicks's[4] paper, putting L_1 for the perpendicular distance between wave-fronts of the light incident on D from the moving source,

$$\frac{\lambda}{L_1} = \frac{4 - 0.5}{4 + 0.5 - 2} = 1.4; \text{ and } \frac{\lambda'}{L_1} = \frac{4-1}{4+1-4} \cdot \frac{4-0.5}{4+0.5+2} = \frac{21}{13}.$$

Therefore

$$\frac{\lambda}{\cos\delta} = L_1 1.4 \div 0.8 = 1.75\ L_1,$$

and

$$\frac{\lambda'}{\cos\delta'} = L_1 \frac{21}{13} \div \frac{12}{13} = 1.75\ L;$$

accordingly,

$$\frac{\lambda'}{\cos\delta'} = \frac{\lambda}{\cos\delta} = l' = l.$$

Therefore, if the intersection of k_{15} and i'_{15} is on the line xy parallel to $q\mathrm{T}_{15}$, the intersection of i_{15}, and h'_{15} is also on the same line; that is, the phase-difference of the two sets of waves is constant along any line parallel to the axis of the observing telescope. The same thing may easily be proved for any one of eight equidistant points of the circumference commencing from the point where the motion of the apparatus is parallel to the axis of the telescope.

The equality is exact for the central ray. For rays inclined as much as five minutes of arc on either side of the central ray,

$$\frac{\cos\delta'}{\lambda'} - \frac{\cos\delta}{\lambda} \leqq 0.2\frac{v^3}{\mathrm{V}^3}.$$

At azimuths other than those specified, the quantity $\frac{\cos\delta'}{\lambda'} - \frac{\cos\delta}{\lambda}$ is not greater than $0.3\frac{v^4}{\mathrm{V}^4}$ for central rays. We may set side by side the magnitudes of this disturbing effect for central rays at several azimuths according to rigorous computation and according to Dr. Hicks's approximate formula.

Azimuths.	Disturbing Effects Compared.	
	Dr. Hicks's formula	Rigorous formula.
90°	$1.2\ v^3/\mathrm{V}^3$	$0.0\ v^4/\mathrm{V}^4$
90° ± 11°	0.9 "	0.3 "
90° ± 22°	0.65 "	0.3 "
90° ± 45°	0.0 "	0.0 "
90° ± 90°	0.0 "	0.0 "

-90° ± 45°	0.0 "	0.0 "
-90°	1.2 "	0.0 "

It will be seen that the effect detected by Dr. Hicks proves, by rigorous computation, to be entirely negligible for the central rays. Its extreme value for marginal rays is not greater than $0.5\frac{v^3}{V^3}$, which is entirely too small to influence the observations. This result is very satisfactory. It is proved for the specified adjustment of the angles, but it is easy to see that the rotation of mirror I about a perpendicular line in its surface, by a quantity like ten seconds of arc, will not change all relations of residual aberrations by important amounts. It is therefore established, at least for the adjustment specified, that the wave-fronts *a* and *a'* of fig. 4. intersect in the line B II, if the wave-fronts *h* and *h"* do, rigorously for eight principal positions, very approximately for all other positions. If, then, we can measure the linear distance between *a* and *a'* at some convenient position T, we may determine the angle between the wave-fronts *a* and *a',* which is the same as the angle between h and *h"*, the angle of aberration of the second degree, which it is the object to detect.

We have shown that the wave-lengths of the two rays, when resolved *in the direction in which alone they are used*, are equal. One other point as to wave-lengths must be considered. We use wave-lengths to measure a length of less than 0.0002 mm., to determine the angle *h B h"*. Is the scale of variable value? The light from a source moving with the apparatus has its wave-length modified by the motion. Dr. Hicks gives the formula for this effect in equation (4), page 17. If with this and the equation (2) we compute the wave-length resolved in the axis, at the azimuths where the effect is a maximum, and for the velocity ratio 100, the two minima are 0.9899995 L and 0.9899505 L, while the two maxima are both 0.9999500 L, where L is the wave-length in the case of no motion. For the ratio 10,000 these quantities differ from unity by about a hundredth part as much, and the inequality is negligible, even if we had to multiply this unit by a large number. But we have to do with only a fraction of the unit.

We next inquire as to the amount and the laws of aberration produced by reflexions from the mirrors of the apparatus. These can be developed in a series of powers of the velocity ratio, and of sines and cosines of the azimuth and of its multiples. But numerical estimates seem desirable, and the formulae are such that these can more easily be obtained from trigonometrical computation. For the *actual* velocity ratio the computation is not easy, because trigonometrical tables of fifteen decimal places are not available. Imagine, then, three different apparatus, each of the dimensions proper to the special value of the velocity ratio for which it is specially designed. One apparatus, for the ratio 10, may have the length B II, fig. 3, equal to 10^2 L; another, for the ratio 100, may have the length 100^2 L; and the third, for the ratio 1000, may have the length 1000^2 L. What we can readily learn for the ratio 10, with seven place logarithms, will apply to the ratio 100, except for the circumstance that angles are not so small that sines and arcs are identical in value. What we compute for the ratio 100 with ten place logarithms tells us everything we desire to know for the ratio 1000 and for the actual ratio.

We have computed the aberrations of the two rays I and II, for certain azimuths with the velocity-ratios 10 and 1000, and for 18 azimuths of the apparatus with the velocity-ratio 100. From these aberrations we subtract that part of the aberration which is annulled by the motion of the telescope, and then decompose these residual aberrations into terms depending on the squares and on the cubes of the ratio of velocities. To a thousandth part of the *residual* aberrations, their difference is represented by the equation

$$\delta' - \delta = \frac{v^2}{V^2}\cos\ 2\alpha + \frac{v^3}{V^3}\left(\sqrt{0.5}\sin\ 2\alpha + \frac{1}{6}\sin\ 4\alpha + \cos\alpha\right).$$

For the velocity-ratio 10,000 the agreement would be much closer. Fig. 9 (Pl. IX.) A shows the laws of the variation in the residual aberrations of the two rays I and II, coming from the mirrors I and II. The unit for A and C is $\sin^{-1}\frac{v^2}{V^2}$, and for B and D, $\sin^{-1}\frac{v^3}{V^3}$. The curves A are nearly represented by

$\frac{v^2}{V^2}\left(\sin\ 2\alpha \pm \frac{1}{2}\cos\ 2\alpha\right)$. Subtracting the ordinates given by this expression from the actual ordinates, we get the residuals shown (after multiplication by the reciprocal of the velocity-ratio) at B. C shows the difference of the curves I. and II. of A, and thus gives directly the angle of divergence of the emergent wave-fronts which is the object of our study. D gives the difference between this curve and the sine curve $\delta' - \delta = \frac{v^2}{V^2}\cos\ 2\alpha$. The latter curve shows that the difference of the aberrations of I and II is at an undisturbed maximum at 90° and at 270°; at 0°, it is less than the undisturbed maximum by the quantity $\frac{v^3}{V^3}\cos\ \alpha$, or $\frac{v^3}{V^3}$; at 180° it is greater by the same quantity.

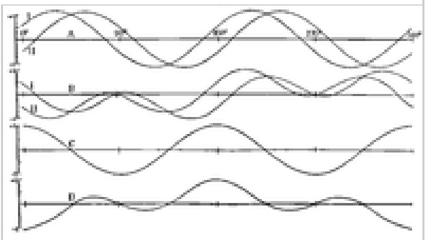

Fig. 9

It may be thought that the adjustment of the angles between the mirrors which has been assumed will limit too narrowly the use of the apparatus. We may simply say that experience with mirrors as nearly plane as those used by us has shown us that the method of observation supposed would suffice for angles of aberration at least twenty-five times that expected if the velocity-ratio is 10,000.

Since the experiment gives a null result, it is not worth the space to prove that what is true of this adjustment is true with sufficient approximation for an adjustment which differs from the assumed adjustment only by the rotation of mirror I by an angle often seconds around a perpendicular axis passing through its surface. Instead, we may compare the results here obtained with those of Dr. Hicks.

In the first place, he declares that the position of the fringes is displaced by aberration. This point is eliminated from our discussion by the fact that the fringes are infinitely wide. We simply remark that, if we understand rightly his statement, this aberration is annulled by the motion of the telescope. Also, his discussion contains a term expressing the fact that the waves of one system gain upon those of the other while passing towards the observer. We have shown that, *in the conditions assumed* (and realized), this effect is *nil* for central rays in the eight principal azimuths, and is small in all others. At its maximum, for central rays, it is $0.3\frac{v^4}{V^4}$. With our present large apparatus, whose length is $54 \times 10^6\lambda$, the gain of one wave-front over the other in the whole length is much less than $10^{-6}\lambda$.

In the theory of 1887, powers of the velocity-ratio higher than the second were expressly regarded as negligible. Dr. Hicks virtually supplies one such term. He writes, displacement of fringes $= \frac{\frac{1}{2}\xi^2 L\cos\ 2\alpha}{\sin(B-A) - \frac{1}{2}\xi^2\cos\ 2\alpha}$, where ξ is the velocity-ratio, L is the length of path in the apparatus, from D to I, fig. 5, and B—A is the difference between the angles DB I and DB II. Without the small term in the denominator, this gives precisely the same value as the expression in the paper of 1887, as a simple numerical computation shows. The effect of the small term is the following: — the value of the denominator is decreased or increased by $\frac{1}{2}\frac{v^2}{V^2}$ at alternate quadrants, and the value of the fraction is therefore increased or decreased at alternate quadrants. But, according to the present solution, the expression should have a mean value at 90° and 270°, and have, further, a maximum at 180° and a minimum at 0°. At three quadrants we agree, but at the fourth we differ by twice the term in question. The difference is easily explained and is negligible, especially in view of the null result of experiment.

It should be noted that, when there is aberration of the wave-front, there are four closely related magnitudes. One is the distance travelled by the wave-front in the period; a second is the perpendicular distance between consecutive wave-fronts, called λ in Dr. Hicks's paper; a third is the distance between wave-fronts, resolved parallel to some line dictated by the geometric conditions of the case; and the fourth is the distance between wave-fronts in the line of sight, which is the true wave-length. The perpendicular distance between wave-fronts is used rightly, as we conceive, in establishing the conditions of the network of intersecting wave-fronts in Dr. Hicks's admirable paper. But in one paragraph, which is entirely distinct from the rest of the discussion, he uses an expression which is not sufficiently approximate; *e. g.*, if the expression be taken to mean the wave-length as stated above, and accordingly used to compute the number of waves in a given length in the line of vision, it differs from the truth by $\frac{v^2}{V^2}$, precisely doubling the result found otherwise.

We assert, then, that the theory of 1887 is correct to terms of the order retained, which were sufficient; that Dr. Hicks's theory agrees with it precisely as to numerical amount and sign[5] of the effect, and that a third examination of the theory gives results differing from those of the two others only by negligible terms of the third order.

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1. Communicated by the Authors. Read at the New York Meeting of the National Academy of Sciences.
2. Am. Jour. Sci. xxxiv. p. 333.
3. Phil. Mag. [6] iii. p. 9 (1902).
4. Phil. Mag. [6] iii. p. 17 (1902).
5. Taking into account a note in 'Nature,' vol. lxv. p. 343 (1902).